Logic – Sample Questions

1. T/F:
   a. $(A \wedge B) \Rightarrow C \; entails \; (A \Rightarrow C) \vee (B \Rightarrow C)$
   b. $(P \wedge \neg R) \Rightarrow (Q \Rightarrow R)$ can be converted into a Horn clause.
   c. $(\forall x \; P(x)) \vee (\forall x \; \neg P(x))$ is a valid sentence.
   d. $\forall x \; x = x$ is satisfiable.
2. Consider $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G)$

   Use resolution to prove that the sentence entails *G*.
3. Correct each logic representation of the following sentences:
   a. "No two people have the same social security number"
      $\neg \exists x, y, n \; Person(x) \wedge Person(y) \Rightarrow (HasSS\#(x,n) \wedge HasSS\#(y,n))$
   b. "John's social security number is the same as Mary's"
      $\exists n \; HasSS\#(John, n) \wedge HasSS\#(Mary, n)$
   c. "Everyone's social security number has 9 digits"
      $\forall x, n \; Person(x) \Rightarrow (HasSS\#(x,n) \wedge Digits(n, 9))$
   d. Rewrite the above sentences (uncorrected) using the function symbols SS# instead of the predicate HasSS#.
4. Translate the following sentences into FOL using the predicates *French, Chilean, Wine, >,* and the functions *Price* and *Quality*:
   a. All French wines cost more than Chilean wines.
   b. The best Chilean wines are better than some French wines.
5. Assume the following propositions: *BatteryDead*, *RadioWorks, OutOfGas,* and *CarStarts*.
   a. What is the total number of models?
   b. How many models are there in which the following sentence is false?
      $(RadioWorks \wedge CarStarts) \Rightarrow (\neg OutOfGas \wedge \neg BatteryDead)$
   c. Is the sentence above equivalent to a set of Horn clauses?
   d. Show that the sentnce above is not entailed by the sentence
      $RadioWorks \Rightarrow \neg BatteryDead$
6. Let *M(x)* be true if x is a mail carrier, *B(x)* is true if x lives in Berkeley, and *K(x,y)* be true if x knows y. Translate the following into FOL:
   a. There are at least two mail carriers who live in Berkeley.
   b. All the mail carriers who live in Berkeley know each other.
7. Consider the following sentence:
   $((Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)) \Rightarrow ((Food \wedge Drinks) \Rightarrow Party)$

a. Determine, using enumeration, whether the sentence is valid, satisfiable or unsatisfiable.
b. Convert the left and right hand sides of the main implication to CNF and verify your answer to a.
c. Use resolution to prove a.

8. Correct the following FOL translations as necessary:
   a. Any apartment in Berkeley has lower rent than some apartments in Palo Alto.
   $$\forall x\left(Apt(x) \wedge In(x, Berkeley\right) \Rightarrow \exists y\left((Apt(y) \wedge In(x, PaloAlto)) \Rightarrow <(Rnt(x), Rnt(y))\right)$$

   b. There is exactly one apartment in Palo Alto with rent below $1000.
   $$\exists x\ Apt(x) \wedge In(x, PaloAlto) \wedge \forall y\left(Apt(y) \wedge In(y, PaloAlto) \wedge <\left(Rnt(y), Dollars(1000)\right)\right) \Rightarrow y = x$$

   c. If an apartment is more expensive than all apartments in Berkeley, it must be in San Francisco
   $$\forall x\ Apt(x) \wedge \left(\forall y\ Apt(y) \wedge In(y, Berkeley) \wedge >\left(Rnt(x), Rnt(y)\right)\right) \Rightarrow In(x, SanFrancisco)$$